5:3a Conservation of Linear Momentum – One Dimensional

All one dimensional linear momentum problems involving two objects can be solved in the same way. The equation describing the momentums of each object before and after a collision is

$$\mathbf{m_1 \cdot v_1 + m_2 \cdot v_2 = m_1 \cdot v_3 + m_2 \cdot v_4}$$

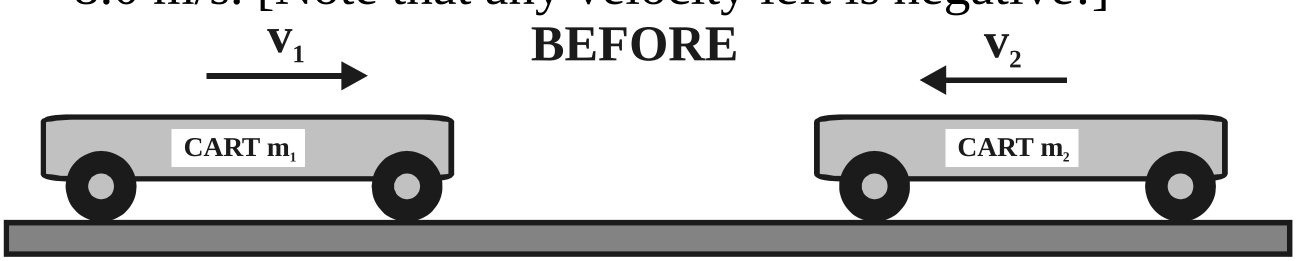
where

- $\mathbf{m_1}$ is the mass of the 1st object
- $\mathbf{v_1}$ is the velocity of the 1st object before the collision
- $\mathbf{v_3}$ is the velocity of the 1st object after the collision
- $\mathbf{m_2}$ is the mass of the 2nd object
- $\mathbf{v_2}$ is the velocity of the 2nd object before the collision
- $\mathbf{v_4}$ is the velocity of the 2nd object after the collision

<u>Inelastic Collisions</u>

The first step here will be to make diagrams showing both objects before as well as after they collide.

For example, suppose that a cart, which has a mass of $\mathbf{m_1}$ = 6.0 kg, is moving toward the right with a velocity of $\mathbf{v_1}$ = 12.0 m/s when it collides with a second cart, which has a mass of $\mathbf{m_2}$ = 4.0 kg, and which is moving toward the left with a velocity of $\mathbf{v_2}$ = -8.0 m/s. [Note that any velocity left is negative!]



The diagram before will show the two carts moving in opposite direction as shown above.

5:3b Conservation of Linear Momentum – Inelastic

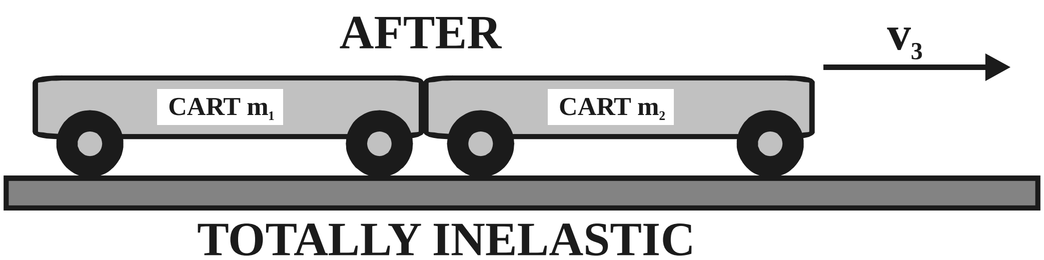
From this information we can calculate the initial total momentum of the system before the collision.

$$\mathbf{p_o = m_1 \cdot v_1 + m_2 \cdot v_2}$$

$$\mathbf{p_o = 6.0kg \cdot 12m/s + 4.0kg \cdot (-8.0m/s)}$$

$$\mathbf{p_o = 72kg \cdot m/s + (-32kg \cdot m/s) = 40kg \cdot m/s}$$

Now suppose that this collision is totally inelastic. This means that the two objects stick together after the collision and that significant amounts of kinetic energy are lost. If so, the diagram after the collision will appear, as shown below.



Since the two carts are attached together after a totally inelastic collision, the final velocities of the carts must be the same and so we can make the final velocities of the carts equal, $\mathbf{v_3 = v_4}$.

$$\mathbf{p_f = m_1 \cdot v_3 + m_2 \cdot v_4 = m_1 \cdot v_3 + m_2 \cdot v_3 = (m_1 + m_2) \cdot v_3}$$

$$\mathbf{p_f = 6.0kg \cdot v_3 + 4.0kg \cdot v_4 = 10.0kg \cdot v_3}$$

Since the total vector momentum **BEFORE** the collision must be equal to the total vector momentum **AFTER** the collision, you merely make the two quantities above equal to one another!

$$\mathbf{p_o = 40kg \cdot m/s = 10.0kg \cdot v_3 = p_f}$$

From which it is easy to solve for the final velocity v_3

$$\mathbf{v_3 = 40kg \cdot m/s / 10.0kg = 4.0m/s}$$

5:3c Conservation of Linear Momentum – Inelastic

Since this is a totally inelastic collision, significant energy will be lost. To determine how much energy was lost in the collision all you need to do is to calculate the total energy of the system before the collision, calculate the total energy of the system after the collision and then take the difference.

Kinetic energy **BEFORE** the collision

$$KE_o = ½\cdot m_1\cdot v_1^2 + ½\cdot m_2\cdot v_2^2$$

$$KE_o = ½\cdot 6.0kg\cdot(12m/s)^2 + ½\cdot 4.0kg\cdot(-8m/s)^2$$

$$KE_o = 432J + 128J = 560J$$

Kinetic energy **AFTER** the collision

$$KE_f = ½\cdot m_1\cdot v_3^2 + ½\cdot m_2\cdot v_3^2$$

$$KE_F = ½\cdot 6.0kg\cdot(4.0m/s)^2 + ½\cdot 4.0kg\cdot(4.0m/s)^2$$

$$KE_F = 48J + 32J = 80J$$

Finally, the energy lost in this inelastic collision is

$$KE_{lost} = KE_o – KE_f = 560J – 80J = 480J$$

Elastic Collisions

Elastic collisions are somewhat more complicated. The diagram before and the calculations before are the same as for inelastic collisions. However, in an elastic collision you need to be concerned about the kinetic energy after the collision, which will be the same both before and after an elastic collision.

5:3d Conservation of Linear Momentum – Elastic

For example, suppose that a cart, which has a mass of $\mathbf{m_1 = 6.0\ kg}$, is moving toward the right with a velocity of $\mathbf{v_1 = 12.0\ m/s}$ when it collides with a second cart, which has a mass of $\mathbf{m_2 = 4.0\ kg}$, and is moving toward the left with a velocity of $\mathbf{v_2 = -8.0\ m/s}$. [Note that any velocity left is negative!]

The diagram before will show the two carts moving in opposite direction as shown above.

From this information we can again calculate the initial total momentum of the system before the collision.

$$\mathbf{p_o = m_1 \cdot v_1 + m_2 \cdot v_2}$$

$$\mathbf{p_o = 6.0kg \cdot 12m/s + 4.0kg \cdot (-8.0m/s)}$$

$$\mathbf{p_o = 72kg \cdot m/s + (-32kg \cdot m/s) = 40kg \cdot m/s}$$

After the collision, however, the carts will **NOT** stick together, but will instead bounce off of one another. How they bounce off of one another will depend on the conditions before the collision. Below is just one example of what might happen.

PERFECTLY ELASTIC

5:3e Conservation of Linear Momentum – Elastic

In this case the momentum after the collision will be given by

$$\mathbf{p_f = m_1 \cdot v_3 + m_2 \cdot v_4}$$

$$\mathbf{p_f = 6.0kg \cdot v_3 + 4.0kg \cdot v_4}$$

This time, however, the final velocities are different and are both unknown. Again you will make the momentum before the collision equal to the momentum after.

$$\mathbf{p_o = m_1 \cdot v_1 + m_2 \cdot v_2 = m_1 \cdot v_3 + m_2 \cdot v_4 = p_f}$$

$$\mathbf{40kg \cdot m/s = 6.0kg \cdot v_3 + 4.0kg \cdot v_4}$$

This time you will notice that there will be two unknown variables $\mathbf{v_3}$ and $\mathbf{v_4}$! So what are you to do? You NEED a second equation and there are two possibilities available; one relatively difficult, the other fairly easy.

The difficult approach is to make use of the fact that this is an elastic collision. This means that the total kinetic energy **BEFORE** the collision should be equal to the total kinetic energy **AFTER** the collision.

Kinetic energy before the collision

$$\mathbf{KE_o = \tfrac{1}{2} \cdot m_1 \cdot v_1^2 + \tfrac{1}{2} \cdot m_2 \cdot v_2^2}$$

$$\mathbf{KE_o = \tfrac{1}{2} \cdot 6.0kg \cdot (12m/s)^2 + \tfrac{1}{2} \cdot 4.0kg \cdot (-8m/s)^2}$$

$$\mathbf{KE_o = 432J + 128J = 560J}$$

Kinetic energy after the collision

$$\mathbf{KE_f = \tfrac{1}{2} \cdot m_1 \cdot v_3^2 + \tfrac{1}{2} \cdot m_2 \cdot v_4^2}$$

$$\mathbf{KE_f = \tfrac{1}{2} \cdot 6.0kg \cdot v_3^2 + \tfrac{1}{2} \cdot 4.0kg \cdot v_4^2}$$

$$\mathbf{KE_f = 3.0kg \cdot v_3^2 + 2.0kg \cdot v_4^2}$$

5:3f Conservation of Linear Momentum – Elastic

Since this is a totally elastic collision the, initial kinetic energy and the final kinetic energy should be equal.

$$\mathbf{560J = 3.0kg \cdot v_3^2 + 2.0kg \cdot v_4^2}$$

Now if you recall the momentum equation from above

$$\mathbf{40kg \cdot m/s = 6.0kg \cdot v_3 + 4.0kg \cdot v_4}$$

you will notice here that we now have two equations and two unknowns. And although this is doable, since it involves the squares in the kinetic energy equation it can become an algebraic pain in the neck! So, what is the alternative?

The alternative is to make use of what is called the **coefficient of elasticity** [or coefficient of restitution - same thing, different name]. The coefficient of elasticity, represented by the letter **e**, relates the **relative velocity** between the two objects **AFTER** the collision to the **relative velocity BEFORE** the collision and is given by the equation

$$\mathbf{e = -(v_4 - v_3)/(v_2 - v_1)}$$

where $\mathbf{v_4 - v_3}$ is the **relative velocity** [remember "relative" means to "take the difference"] between the two objects **after** the collision and where $\mathbf{v_2 - v_1}$ is the **relative velocity** between the two objects **before** the collision. The negative sign is there because the objects reverse direction when they bounce off of one another. The coefficient of elasticity will be exactly **e = 1** if the collision is elastic, **e < 1** if the collision is inelastic [in fact **e** = **exactly 0** if the collision is **perfectly inelastic**!] and will be **e > 1** if the interaction is an **explosion**.

5:3g Conservation of Linear Momentum – Elastic

Solving the equation for coefficient of elasticity and the equation for momentum conservation simultaneously is a **MUCH** easier task than solving the equation for momentum conservation simultaneously with the equation for energy conservation as you can see below!

$$\mathbf{e = -(v_4 - v_3)/(v_2 - v_1)}$$

$$\mathbf{1 = -(v_4 - v_3)/(-8m/sec - 12m/sec) = -(v_4 - v_3)/(-20m/sec)}$$

$$\mathbf{20m/s = v_4 - v_3}$$

If you solve this for v_4 you will get

$$\mathbf{v_4 = 20m/s + v_3}$$

This expression can then be used to solve for v_3. Before

$$\mathbf{p_o = m_1 \cdot v_1 + m_2 \cdot v_2}$$

$$\mathbf{p_o = 6.0kg \cdot 12m/sec + 4.0kg \cdot (-8.0m/sec) = 40kg \cdot m/s}$$

and then substitute the above expression after

$$\mathbf{p_f = m_1 \cdot v_3 + m_2 \cdot v_4}$$

$$\mathbf{p_f = 6.0kg \cdot v_3 + 4.0 \cdot v_4 = 6.0kg \cdot v_3 + 4.0kg \cdot (20m/s + v_3)}$$

make the momentum before, equal to the momentum after

$$\mathbf{40kg \cdot m/s = 6.0kg \cdot v_3 + 4.0kg \cdot (20m/s + v_3)}$$

and then solve for v_3

$$\mathbf{40 = 6v_3 + 80 + 4v_3} \text{ leading to } \mathbf{-40 = 10 \cdot v_3}$$

$$\mathbf{v_3 = -40/10 = -4.0m/sec}$$

5:3h Conservation of Linear Momentum – Elastic

Finally, just substitute the value you got for $\mathbf{v_3}$ back into the equation from above and solve for $\mathbf{v_4}$!

$$\mathbf{v_4 = 20 + v_3 = 20m/sec + -4.0m/sec = 16m/sec \quad Done!}$$

One very strong hint! After solving for $\mathbf{v_3}$ and $\mathbf{v_4}$ always substitute back into the original momentum conservation equation to make sure your answers really are correct!

$$\mathbf{p_o = m_1 \cdot v_1 + m_2 \cdot v_2 = m_1 \cdot v_3 + m_2 \cdot v_4 = p_f}$$

The total linear momentum before

$$\mathbf{p_o = m_1 v_1 + m_2 v_2}$$

$$\mathbf{p_o = 6.0\ kg \cdot (12\ m/sec) + 4.0\ kg \cdot (-8.0\ m/sec)}$$

$$\mathbf{p_o = 72kg \cdot m/sec - 32kg \cdot m/sec =}$$

$$\mathbf{p_o = 40kg \cdot m/s}$$

The total momentum after

$$\mathbf{p_f = m_1 v_3 + m_2 v_4}$$

$$\mathbf{p_f = 6.0kg \cdot (-4.0m/sec) + 4.0\ kg \cdot (16\ m/sec)}$$

$$\mathbf{p_f = -24kg \cdot m/sec + 64kg \cdot m/sec}$$

$$\mathbf{p_f = 40kg \cdot m/sec}$$

And as you can see, the initial and final momentums are equal!