Unit 8 Lesson 1: Inverses, Inverse Sine & Compositions

Inverse Sine Function:

Opening Activity – Review Inverse Functions

Find the inverse of the following linear function:
Remember that domains and ranges switch from an original relation/function to its inverse (x and y switch).
Also remember from Unit 5 that inverse relations/functions are symmetric over the line y=x.

$$y = f(x) = \frac{4x-1}{3}$$

Method 1:

$$y = f(x) = \frac{4x-1}{3}$$

$\downarrow \bullet 4 \quad \uparrow \div 4 \Rightarrow \frac{3x+1}{4} \qquad \therefore f^{-1}(x) = \frac{3x+1}{4}$

$\downarrow -1 \quad \uparrow +1 \Rightarrow 3x+1$

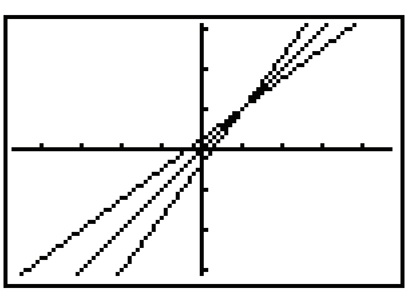
$\downarrow \div 3 \quad \uparrow \bullet 3 \Rightarrow 3x$

Method 2:

$$y = \frac{4x-1}{3} \Rightarrow \text{Switch y with x and solve for y}$$

$$x = \frac{4y-1}{3} \Rightarrow 3x = 4y-1 \Rightarrow 4y = 3x+1 \Rightarrow y^{-1} = \frac{3x+1}{4}$$

Observe the calculator graph of the linear function and its inverse function below:



The middle line is the graph f(x)=x → y=x.

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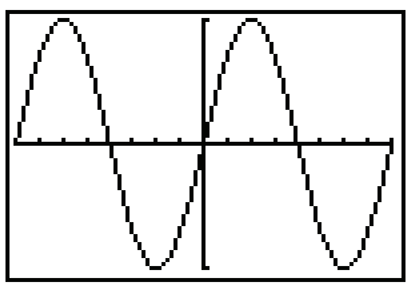
Find $(f \circ f^{-1})(x)$ and $(f^{-1} \circ f)(x)$ for the sample problem:

$$\left(f \circ f^{-1}\right)(x) = \frac{4\left(\frac{3x+1}{4}\right)-1}{3} = x$$

$$\left(f^{-1} \circ f\right)(x) = \frac{3\left(\frac{4x-1}{3}\right)+1}{4} = x$$

Inverse Sine:

Graph the sine function on your graphics calculator using the range -2π to 2π



Is it a function?
Answer: Yes, because it passes the vertical line test and one element in the domain does not map to more than one element in the range.

Is it a one-to-one function?
Answer: No, because it fails the horizontal line test and its inverse is not a function.

Could we restrict the domain of the sine function to form a region that is one-to-one?

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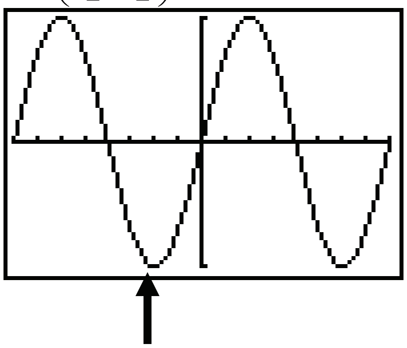
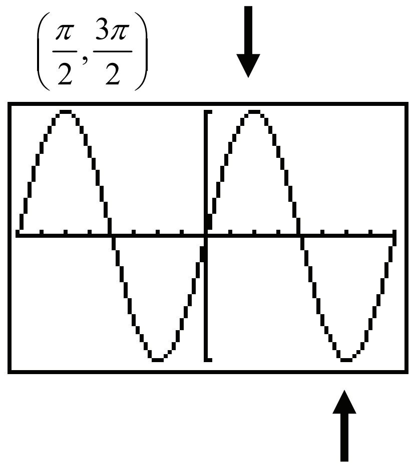
Observe various correct choices:

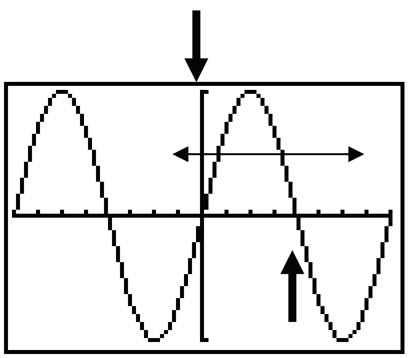
$\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$



$\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$



What is the restricted region closest to the origin (yet avoids as many negative values as possible) that allows the inverse to become a function?
Try starting at the origin:



If you start at the origin, the section formed fails the horizontal line test and is not a function!

Inverse Sine is defined as follows:

The best way to confine the sine function (that is closest to the origin) is between

$$\frac{-\pi}{2} \text{ and } \frac{\pi}{2}.$$

Therefore:

$$y = \sin^{-1} x \text{ or } y = \text{Arcsin}(x) \text{ iff } \sin y = x \text{ and } \frac{-\pi}{2} \le y \le \frac{\pi}{2} \text{ and } -1 \le x \le 1$$

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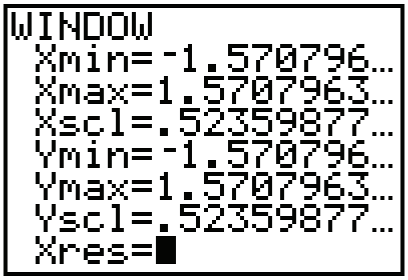
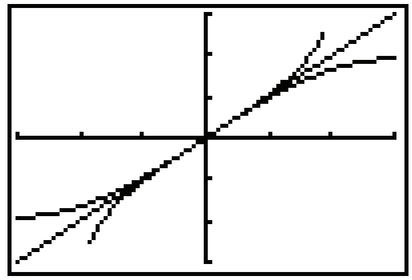
Observe the graph of sine, inverse sine and y=x:





Refer to "Handout Unit – 8 Lesson 1".

Arcsin Exact Values*:*