Unit 4:Polynomial and Rational Functions: Lessons 1-6

I. Unit Objectives

- Students will define a polynomial and divide polynomials.
- Students will apply the Remainder Theorem, the Factor Theorem, and the connections between remainders and factors.
- Students will determine the maximum number of zeros of a polynomial.
- Students will find all rational zeros of a polynomial function using the Factor Theorem.
- Students will factor a polynomial completely.
- Students will find lower and upper bounds of zeros.
- Students will recognize the shape of basic polynomial functions and will describe their graph.
- Students will identify properties of general polynomial functions: continuity, end behavior, intercepts, local extrema, and points of inflection.
- Students will identify complete graphs of polynomial functions.
- Students will fit a polynomial model to data.
- Students will find the characteristics of a rational function: domain, intercepts, vertical asymptotes, horizontal asymptotes, holes, and end behavior.
- Students will sketch complete graphs.
- Students will write complex numbers in standard form, perform arithmetic operations on complex numbers, and find the conjugate of a complex number.
- Students will simplify square roots of negative numbers.
- Students will find all solutions of polynomial functions.
- Students will use the Fundamental Theorem of Algebra.
- Students will find complex conjugate zeros and find the number of zeros of a polynomial.
- Students will give the complete factorization of polynomial expressions.

Unit 4: Lesson 1 – Polynomial Functions

Lesson Objectives:

- Students will define a polynomial.
- Students will divide polynomials.
- Students will apply the Remainder Theorem, the Factor Theorem, and the connections between remainders and factors.
- Students will determine the maximum number of zeros of a polynomial.

Teaching Strategies and Content Background:

4.1a: Open the lesson with an example like 143 divided by 4. Ask the students to identify the quotient, divisor, dividend, and remainder. The definitions are important to build a common language of terms when working with polynomials.

4.1b: Completing polynomial division with the long division method is tedious and most students find it difficult. The teacher should make connections between doing long division with the first example and the process of doing polynomial long division. Synthetic division should only be used when the divisor is a first-degree polynomial, else the long division method is appropriate. Students often get this confused when trying to find "short cuts" for the division problems.

4.1c: The teacher should reinforce the steps for solving polynomial synthetic division when working examples. The students will misunderstand what number goes into the half-box of a synthetic division. It depends if a possible root is given (then you use that number directly), or if a possible factor is given (then you use the opposite sign before the number).

4.1d: The Remainder Theorem is very important to emphasize when finding the zeros of a polynomial. It is good to show by example how the Remainder Theorem works to find zeros of the polynomial.

4.1e: It is important to show that if a binomial can be shown to be a factor of a polynomial, then the binomial is also a zero of the polynomial function.

4.1f: Conceptually, the fundamental polynomial connections is extremely important for the students to see how finding zeros, x-intercepts, solutions, roots, and factors of a polynomial are all related to each other. Students often get confused about finding the linear factors of a polynomial, so the teacher should take time to explain how to find the linear factors.

4.1g: The teacher can close the lesson with "Give five ways to show that x – 3 is a factor of $f(x) = x^2 + 3x - 18$". The answers are: (1) factor $x^2 + 3x - 18$ into (x – 3)(x + 6), (2) Use long division of $x^2 + 3x - 18$ by x – 3 to show the remainder of 0, (3) Use synthetic division to show the remainder of 0, (4) Show f(3) = 0 by the Factor Theorem, and (5) Graph the function to show that 3 is an x-intercept.

Unit 4: Lesson 1 – Polynomial Functions

Frequently Asked Questions:

Why in synthetic division when we divide by x – 2 that we have a positive 2 as our divisor? Why is it the opposite sign?

When completing polynomial division by synthetic division, we are testing the possible zero of the polynomial. In the example with a divisor of x – 2, replacing x with positive 2 would make the divisor equal to zero, so positive 2 must be used in the half-box.

In a synthetic division example on SW 4.3, how do you know that the first coefficient 3 goes with the term x^3*?*

When you get the numbers on the bottom of the synthetic division algorithm, you start with the right most number and put a box around that number (which is the remainder). Then the number to the left of the boxed number is the constant term, the number to the left of the constant term is the x term, and so on.

<u>Assessment:</u>

Assign 10 homework problems for polynomial functions from your textbook.

Unit 4: Lesson 2 – Real Zeros

<u>Lesson Objectives:</u>

- Students will find all rational zeros of a polynomial function.
- Students will use the Factor Theorem.
- Students will factor a polynomial completely.
- Students will find lower and upper bounds of zeros.

<u>Motivation/Opening Activity:</u>
Use some form of "bell work" to check the students' understanding of Lesson 1. Ask the students, "If 2 was a real zero of $x^3 - 4x^2 + x + 6$, how could you find the other exact zeros without graphing?" The answer is use synthetic division to find the resulting quadratic equation, then factor the quadratic equation to find the other 2 zeros.

<u>Teaching Strategies and Content Background:</u>

4.2a: Using the rational zero test, students often confuse the factors of r (for the constant term) and s (for the leading coefficient). A listing of all possible zeros is the possible combinations the fraction r divided by s.

4.2b: finding the x-intercepts of the function graphically can approximate Finding the zeros and the Factor Theorem.

4.2c: Students can use the graphing method to find exact zeros of the function, then use those exact zeros to use synthetic division to find a resulting quadratic equation, and finally to factor the quadratic equation to find all the zeros of the function.

4.2d: The bounds test is a method to test whether a function has a zero in a beyond a certain x-value.

4.2e: Using the four-step approach to find real zeros of polynomials is a good way to find exact zero values. Otherwise, the graphing approach gives only approximately values. The teacher can close the lesson by asking, "What steps would you use to go about finding the exact real zeros of a polynomial?" This should be a reinforcement of the four-step method shown.

Frequently Asked Questions:

What is the difference between a zero and a factor of a polynomial?

There is no difference. A zero represents an x-value where the polynomial is 0 and a factor is in the form (x – a) where a is the zero of a polynomial.

Unit 4: Lesson 2 – Real Zeros

What is the difference between a zero and an x-intercept?

There is no difference between a zero (algebraically) and an x-intercept (graphically).

Are there n real zeros for an nth degree polynomial?

For an nth degree polynomial, there are n zeros, which can be either real zeros or non-real zeros.

<u>Assessment:</u>

Assign 10 homework problems for real zeros from your textbook.