Unit 4 Geometry

Unit Overview:

Regardless of whether geometry developed from a practical need whilst surveying and constructing or from an artistic need for balance and pattern it is clear that the rudiments of geometry were well known before the times of even the earliest civilisations. Pottery, weaving, basketry, even some pictures painted by stone-age man, show an appreciation of congruence, symmetry and other properties of shape. By the time of the Greeks, number and geometry were almost inseparable. In the 17th century, Descartes suggested a marriage between geometry and algebra which eventually led to the development of calculus and the mathematics of today.

Following this same broad sequence this unit starts with some basic concepts of shape and space. Pythagoras' rule, the most famous of theorems, is proved using geometric principles. This same rule is then used within the realm of coordinate geometry. Finally a study of straight lines and slopes acts as foundation upon which to build the differential calculus met in unit 8.

Lesson 4:1 2D Shapes and Properties

Curriculum Objective(s):

- A mini-project style piece of work involving the generation of, analysis and evaluation of data

Students' Background Knowledge
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It is expected that students should already be familiar with the general concepts contained in this lesson.

Ideas from this lesson will later be extended in lesson 5:10 '3D shapes and properties'.
One aim of the project is for students to find a way of extending the task. This is a very important skill to develop in order for a full response to the coursework component to be made.

Materials: Paper and scissors

Procedure:

No particular introduction is necessary. Students should work in small groups/pairs whilst in class. The students should start the investigation in class, and be given about one week to write up their results as a mini-coursework style project.

Time can be left towards the end of the lesson for students to share their findings so far, but more importantly to discuss possible extensions to the project. If students are struggling with this, the following could be suggested as good extension ideas:

- Investigating the (line and rotational) symmetrical properties of the shapes.
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- Repeating the investigation with different starting shapes (say an equilateral triangle).
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- Categorising shapes efficiently by their properties (see '20 questions game' below).

It is also useful to talk generally about how the project should be written up, and how it will be assessed (see TW 2:1 for notes on this). If possible, before the final submission of this project, allow students some additional time in class to discuss possible ways of extending the project, sharing any problems they may have encountered.

There are two games that follow on quite naturally from this investigation that can be fun to play and that reinforce ideas from this lesson. They can be used at the end of this lesson or as lesson-fillers in future lessons. (Ideas taken from *Mathematical Imagery, School Mathematics Project*)

20 Questions – Ask a group to select a shape. Another group must ask questions that can only be answered with a yes, no, or not applicable. Other students can also be told the name of the shape and act as 'referees' to endure fair play. The object of the game is to deduce the shape through the least amount of questions. This is a very simple and enjoyable game to play that encourages precise communication and logical thought.

Clock Shapes – Describe shapes using the 12 points of clock. For example; 12, 3, 6, 9 is a square. Ask students to give the points for another square. Now a rectangle, trapezium, isosceles triangle… This activity work best if all clock faces and watches in the room can be hidden or removed.

Student Activity: Ex 4:1 from TW

Follow up Lesson:

Ask students to present their findings to the class, either individually or in groups. Give students constructive feedback on their written reports. Allow students time to discuss other students contributions to the aspect of extending the project. Students should feel that their contributions are valid, but that some ideas are going to be more fruitful than others.

Lesson 4:2 Area

Curriculum Objective(s):
Presumed knowledge: – simple two-dimensional shapes and their properties, including perimeters and areas of circles, triangles, quadrilaterals and compound shapes. The general formula for the area of a triangle ($\text{Area} = \frac{1}{2}ab\sin C$ *) is not treated in this lesson, but is covered in lesson 6:6.*

Students' Background Knowledge:

It is expected that students should already be familiar with the general concepts contained in this lesson.

However students are probably less aware of how the shapes and their related area formulae are connected.

Materials: Thread or string for measuring the circumference of a circle.

Procedure:

The aim of this lesson is to reinforce students' previous knowledge of area through practical demonstration. By stressing the relationships between common shapes, the connections between related formulae should become more tangible.

The lesson should be split into two parts. The **first part** deals with general concepts and areas of simple plane shapes. The **second part** reviews previous knowledge of circles.

It is worth leaving time to start the second part of the lesson in class, even if the first part has not been finished. It is unlikely that students will finish the assignments in class but instead finish the activities in their own time.

Start the lesson by asking the students to define 'area'. This tends to be not as simple a question as it sounds, and the discussion can take several false directions before students will agree on a definition similar to 'the amount of space a (flat) shape covers'.

Draw two irregular shapes on the board and ask which has the larger area. Focus on asking students how they could prove or justify their answer. Guide the discussion towards the idea of 'counting squares' inside the shapes.

Use the PowerPoint slides 3:2 and the notes below.

Slide a
Rectangles
To find the area of a rectangle we simply count the squares inside, and a fast way of counting the squares is to multiply the length by the width. It is important to stress that the length and width must be perpendicular to each other.

Slide b
Triangles
All triangles can be thought of as half rectangles. Again, stress that the key measurements are at right-angles to each other.

Slide c
Parallelograms and Kites
Once again, the key measurements are at right-angles to each other.

Slide d
Trapeziums
The diagrams show how a trapezium can be split into two triangles.

Student Activity: Ex 4:2a from TW

For the second part of the lesson ask students to recall formulae they have used related to circles. It is likely that students are able to recall the formulae correctly, but it is interesting to hear their definitions of π. The worksheet 3:2b provides a practical activity to reinforce the concept of π and some simple questions on areas and circumferences of circles.

Student Activity: Ex 4:2b from TW

Background Support Notes:

π
It is useful at this stage to discuss students perceptions of π, reminding students that it represents the ratio of the circumference of *any* circle to its diameter. It is interesting to reflect that π is a *universal* constant, and that the concept of π has been around since antiquity. For example it is known that Babylonian mathematicians from as early as the 20th century BC used a value of 25/8 to approximate π.

It is because π is an irrational number and thus impossible to write down as an exact decimal expansion that it is represented by a special symbol.

From two of the more common approximations of the value of π come the following trivia:
In the US, March 14th is *National Pi Day* (3.14 in US date format).

Whilst in Europe, July 22nd is celebrated as *Pi Approximation Day* (22/7 in Euro date format)

Some students confuse the formulae for circumference and area. For this reason it is worth pointing out that area is measured in squares and the formula for the area of a circle is $\text{Area} = \pi r^2$, with the emphasis on the r^2 term having dimensions $[\text{Length}] \times [\text{Length}]$.

Circumference, πd or $2\pi r$, is simply a measure of length, having dimensions (number)×[Length].

These formulae are not included in the information (formula) booklet that accompanies the course and so must be learnt.

Lesson 4:3 Pythagoras' Theorem

Curriculum Objective(s):

- Presumed knowledge – Pythagoras' theorem

Students' Background Knowledge:

It is expected that students should already be familiar with the general concepts contained in this lesson.

Though students tend to have good recall of this theorem, the geometric proof in the student exercise tends to be well received.

Procedure:

A short discussion about Pythagoras and the discovery of the theorem is an excellent opportunity to bring in the rich multicultural history of mathematics.
Student activity Ex 4:3 from TW includes a geometric proof, out of well over a hundred that are known, of Pythagoras' theorem.

The proof is for enjoyment and understanding only and certainly not required to be reproduced by students taking this course.

Less familiar to most students is the converse form of Pythagoras' theorem, that the existence of a right angle in a triangle can be proved by considering the lengths of the sides. This is addressed in the student exercises, question 2, and is well worth highlighting.

Student Activity: Ex 4:3 from TW

Background Support Notes:

Pythagoras ($\approx$580-540BC)

Pythagoras is probably the most famous of all mathematicians and the theorem attributed to him is arguably the most well known in mathematics.

Pythagoras was certainly not the first mathematician to discover this rule. Babylonian clay tablets dating from around 1500BC imply knowledge of the theorem. Chinese texts from 500BC also cite particular instances of the rule which are certain to have been derived independently. There is also evidence that Egyptian and Arab mathematicians knew of the rule well before its treatment by Pythagoras. However the first *general* result and rigorous proof of the theorem is attributed to the Pythagorean School and is the principal reason the theorem takes his name.

Further Biographical information about Pythagoras can be found at
http://en.wikipedia.org/wiki/Pythagoras

Lesson 4:4 Midpoint, Distance between 2 points

Curriculum Objective(s):

- Coordinates in two dimensions: points; lines; midpoints
- Distances between points

Students' Background Knowledge:

For some students the generalised formulae derived from this lesson will be new. Even for those with prior knowledge the investigative approach adopted here should ensure a more concrete understanding.