7:11a Integration – Substitution Rule

Basic Integration

Evaluate the following expressions.

a) $\int(\sin x)\,dx$

$= -\cos x + C$

b) $\int(e^x)\,dx$

$= e^x + C$

c) $\int(3x^2 + \frac{1}{x})\,dx$

$= x^3 + \ln x + C$

d) $\int(3\frac{1}{\cos^2 x})\,dx$

$= 3\tan x + C$

7:11b Integration – Substitution Rule

Consider the following …

Differentiate y = √(x² +1).

$$\frac{d}{dx}\sqrt{x^2+1} = \frac{d}{dx}\left(x^2+1\right)^{\frac{1}{2}}$$

$$= \frac{1}{2}\left(x^2+1\right)^{-\frac{1}{2}}(2x)$$

$$= x\left(x^2+1\right)^{-\frac{1}{2}}$$

Thus, the anti-derivative of $y = x(x^2 + 1)^{-1/2}$ is

$$\int x(x^2+1)^{-\frac{1}{2}} = \sqrt{x^2+1} + C$$

7:11c Integration – Substitution Rule

Substitution Rule

What we are interested in doing now is how to "undo" the chain rule differentiation rule.

We know that $\frac{d}{dx}F(g(x)) = F'(g(x)) \cdot g'(x)$

Chain Rule

Then, $\int \left[F'(g(x)) \cdot g'(x)\right] dx = F(g(x)) + C$

Undoing the chain rule.

7:11d Integration – Substitution Rule

Instead of using g(x), let us substitute u = g(x).

If u = g(x), $g'(x) = \frac{du}{dx}$

Then $\int \left[F'(g(x)) \cdot g'(x)\right] dx = F(g(x)) + C$

Becomes $\int \left[F'(u)\frac{du}{dx}\right] dx = F(u) + C$

$$\int \left[F'(u)\, du\right] = F(u) + C$$

7:11e Integration – Substitution Rule

Example 1

Evaluate $\int \left[(x^3+3)^4(3x^2)\right]dx$

We need to find a g(x) such that we can substitute g(x) = u and $g'(x) = \frac{du}{dx}$.

We see that if u = x^3 + 3 then $\frac{du}{dx} = 3x^2$

or $du = 3x^2 dx$

$$\int \left[\underbrace{(x^3+3)}_{u}{}^4 \underbrace{(3x^2)\,dx}_{du}\right]$$

7:11f Integration – Substitution Rule

$$\int \left[\underbrace{(x^3+3)}_{u}{}^4 \underbrace{(3x^2)\,dx}_{du}\right]$$

The substitution yields $\int u^4 du$

$$\int u^4 du = \frac{u^5}{5} + C$$

Replacing u = x^3 + 3,

$$\int \left[(x^3+3)^4(3x^2)\right]dx = \boxed{\frac{(x^3+3)^5}{5} + C}$$