Print Name: ______________________ Period: _____________ Date: _____________

CALCULUS

Derivative of Natural Log and Exponential Functions

Given: $e \equiv \lim_{n\to 0}(1+n)^{\frac{1}{n}}$ and that the function $f(x) = \ln(x)$ is the continuous inverse of the function $g(x) = e^x$. Thus ln has all the properties known from PreCalculus.

Exponential Function	**Logarithmic Function**
$y = e^x$	$y = \ln(x) \iff e^y = x$
Domain $= \mathbb{R}$	Domain $= \{x > 0\}$
Range $= \{y > 0\}$	Range $= \mathbb{R}$
Continuous	Continuous
$a^n \cdot a^m = a^{n+m}$	$\ln(A) + \ln(B) = \ln(A \cdot B)$
$a^n \div a^m = a^{n-m}$	$\ln(A) - \ln(B) = \ln\left(\frac{A}{B}\right)$
$(a^n)^m = a^{n\cdot m}$	$k \cdot \ln(A) = \ln\left(A^k\right)$

Derivative of $\ln x$

$$\frac{d}{dx}\ln x = \lim_{h\to 0}\frac{\ln(x+h)-\ln(x)}{h}$$

$$= \lim_{h\to 0}\left[\frac{1}{h}\ln\left(\frac{x+h}{x}\right)\right]$$

$$= \lim_{h\to 0}\left[\frac{1}{h}\ln\left(1+\frac{h}{x}\right)\right]$$

Let $n = \frac{h}{x}$ thus $h = nx$

$$\frac{d}{dx}\ln x = \lim_{n\to 0}\left[\frac{1}{nx}\ln(1+n)\right]$$

$$= \frac{1}{x}\lim_{n\to 0}\left[\ln(1+n)^{1/n}\right]$$

$$= \frac{1}{x}\ln\left[\lim_{n\to 0}(1+n)^{1/n}\right]$$

$$= \frac{1}{x}\ln(e)$$

$$= \frac{1}{x}$$

$$\boxed{\frac{d}{dx}\ln x = \frac{1}{x}}$$

Derivative of e^x:

$$y = e^x$$

$\ln(y) = x$ take the derivative implicitly

$$\frac{1}{y}\cdot\frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = y$$

$$\frac{dy}{dx} = e^x$$

Now to evaluate the derivative of a^x for other basis use the fact that $a = e^{\ln(a)}$ and apply the chain rule.

$$\frac{d}{dx}a^x = \frac{d}{dx}\left(e^{\ln(a)}\right)^x$$

$$= \frac{d}{dx}\left(e^{x\cdot\ln(a)}\right)$$

$$= \left(e^{x\cdot\ln(a)}\right)\cdot\ln(a)$$

$$= a^x\cdot\ln(a)$$

$$\boxed{\frac{d}{dx}a^x = a^x\cdot\ln(a)}$$