Unit 5:1A Introduction to Implicit Derivative

Leibniz Notation:

$$\frac{df}{dx} = \frac{df}{du} \cdot \frac{du}{dx}$$

Chain Rule:

Let $y = (\ )^3$, then $\frac{dy}{dx} = 3(\ )^2 \frac{d}{dx}(\ )$.

$y = (\cos(x))^3$, then $\frac{dy}{dx} = 3(\cos(x))^2 \frac{d}{dx}(\cos(x))$.

$y = (u)^3$, then $\frac{dy}{dx} = 3(u)^2 \frac{d}{dx}(u)$.

The examples above are defined explicitly.

In a sense we took the $\frac{d}{dx}$ of both sides of the equation.

5: 1B

Example:

Let $y = \frac{1}{x}$, then $\frac{dy}{dx} = \frac{d}{dx} x^{-1} = -x^{-2} = -\frac{1}{x^2}$.

$y \cdot x = 1$, this is called an *implicit* relationship.

The derivative $\frac{dy}{dx}$, can be found by differentiating both sides of the implicit relationship.

$$\frac{d}{dx}(y \cdot x) = \frac{d}{dx}(1),$$

$$\frac{d}{dx}(y) \cdot x + y\frac{d}{dx}(x) = 0,$$

$$\frac{dy}{dx}x + y = 0$$

$$\frac{dy}{dx} = -\frac{y}{x}$$

Since $y = \frac{1}{x}$, the two results for $\frac{dy}{dx}$, are equivalent.

$$-\frac{1}{x^2} = -\frac{y}{x}$$

5: 1C

Vertical: When is the tangent line to the curve $y \cdot x + 3x^2 = 408$ vertical?

$$\frac{dy}{dx} \cdot x + y \cdot 1 + 6y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{y}{x+6y}$$

Slope of tangent line is vertical when $x = -6y$

Substitute back into original equation:

$$y(-6y) + 3(-6y)^2 = 408$$

$$-6y^2 + 108y^2 = 408$$

$$102y^2 = 408$$

$$y^2 = 4$$

$$y = \pm 2$$

Points of vertical tangency occur at (12,-2) and (12,2).

Horizontal: When is the tangent line to the curve $y \cdot x + 3x^2 = 408$, horizontal?

Slope of tangent line is $\frac{dy}{dx} = -\frac{y}{x+6y}$ and thus horizontal when y=0

Substitute back into original equation:

$$0 \cdot x + 3x^2 = 408$$

$$x^2 = 136$$

$$x = \pm\sqrt{136}$$

Points of horizontal tangency occur at $\left(\sqrt{136},0\right)$ and $-\left(\sqrt{136},0\right)$.