Unit 4: Limits and Differentiation

I. Unit Objectives
- The student will gain an informal understanding of limits from a numerical, graphical and analytical perspective.
- The student will know both the formal definition and concept of continuity.
- The student will be able to analyze the continuity of a function over a given region.
- The student will be able to analyze the continuity of piecewise functions.
- The student will understand differentiation in the context of tangents and in terms of rates of change.
- The student will learn techniques of calculating derivatives and why they are true.
- The student will find the limit of algebraic and trigonometric expressions using analytical techniques.
  - The student will be able to take the derivative of polynomials, products, quotients, and simple trigonometric expressions.
  - The student will be able to take the derivative of piecewise functions.
  - The student will understand the relationship between the derivative and regions of increase and decrease in of the graph a function.
  - The student will understand the derivative of a position function as it relates to velocity, acceleration and speed.
  - The student will be able to take the derivative of composition functions using the chain rule.

Unit Overview:
At our school the first three units are included in the spring of the prerequisite course: "Introduction to Calculus." This unit is a review of this material. This unit may be omitted unless you are in a similar situation. This review will clarify the fundamental ideas of limits and differentiation. Students should come to understand that it is the concept of limits that defines what mathematics is called Calculus. Students will apply this understanding to various problems from the text, on quizzes and take home assignments.

II. Unit Lessons:

Lesson 1 – Intro to Class and Review of Limits

<u>Lesson Objectives:</u>
- The student will gain an informal understanding of limits from a numerical, graphical and analytical perspective.

<u>Material List:</u>
- Course Syllabus and Homework Sheet.
- Worksheet 2.1 – "Review of Limits"

<u>Procedure</u>: Introduction to the course, discuss the history and etymology of "Calculus" and review idea of limits.

<u>Description:</u>
Start by asking students if they have any ideas about calculus, or what calculus is all about. The typical method of starting most any course is to ask students what they hope to gain from this course.

Stress that calculus is very easy. Few students have trouble with calculus. The difficulty experienced by students in a calculus class has little to do with their knowledge of calculus. The trouble is most often with algebra. To apply calculus ideas, a student must be very very good at algebra. Graphing, factoring, and manipulation will be a small required part of much larger more complicated problems. Stress to the students that as they struggle, be sure to distinguish between struggling with the concepts and big ideas or simply struggling with the algebra. The big ideas are the important part but to be successful in this course they must be meticulous in their algebraic work.

Introduce students to course syllabus and standards. Handouts on the course including such topics as grade standards and course policies are typically viewed legally as a contract between you and your students, so be clear and encourage questions.

Introduce the concept of limits. Use the review of limits worksheet Discuss the idea of direct substitution and when that goes wrong (0/0).

<u>Assessment:</u>
- Assign a fifteen or so problems from the text that review the basic ideas of limits.

Lesson 2 – Continuity

Lesson Objectives:
- Know both the formal definition and concept of continuity.
- Analyze the continuity of a function over a given region.
- Analyze the continuity of piecewise functions.

Material List:
- Continuity Worksheet (2.2)

Procedure: Review homework, review previous topics, introduce new topics, and present examples.

Description:
Check homework assignment and discuss selected problems as requested by students. Be certain to review problems #15 and 21 from the homework. Ask about what it means to be continuous – the concept of no breaks, drawing without lifting the pen.

Discuss the three-part definition of continuity.   Some text only list the third part $\left(\lim_{x \to a} f(x) = f(a)\right)$, but this implies that the function and limit exist at $x=a$. Give examples of functions that satisfy two parts of the definition but not all three parts. Talk about the continuity of piecewise functions. Finding points of discontinuity is like finding holes in your pants – check each piece of material and each seam. Have students example in class notes. Use the continuity worksheet to review continuity.

Assessment:
- Assign a fifteen or so problems from the text that review the basic ideas of continuity.