7.1 Definition of Polynomials

Degree – of a <u>variable</u> in a monomial, the number of times the variable occurs as a factor in the monomial.

$7x^2y$ has degree 2 for x, degree 1 for y

Monomial – a constant, variable, or a product of a constant and one or more variables.

.3, x, 4y, -tz

7.1 Definition of Polynomials

Polynomial – a monomial or a sum of monomials. The monomials are called the *terms* of the polynomial.

$3x^2y$

Similar monomials – (aka: like terms) are identical in their variables and degree, only different in coefficients.

5xy and –2xy

7:1 Definition of Polynomials

When you add or subtract polynomials, you add or subtract the coefficients of terms with the same degree, or like terms.

Example 1.* (3x + 5x) = (3+5)x = 8x

$$*\ (6x^2+5x+2)+(3x^2+2x+5)$$
$$(6+3)x^2+(5+2)x+(2+5)$$
$$9x^2+7x+7$$

7:1 Definition of Polynomials

Example 2:* (x+7) – (3x+2)

x + 7 – 3x – 2 = (1-3)x + (7-2)

-2x – 5

$$*\ (-6x^2+10x-3)-(4x^2-7x)$$
$$(-6+4)x^2+(10-7)x-3$$
$$-10x^2+3x-3$$

7:1 Definition of Polynomials

To multiply two binomials (polynomials with two terms) mentally…FOIL method.

The FOIL method is an acronym to help you remember to multiply two binomials by including each term. The letters of FOIL represent " First - Outside - Inside - Last", or the order in which to multiply using the distributive property. Let's see an example of what this involves:

7:1 Definition of Polynomials

Example 3: Multiply (3x + 2)(2x + 1)

3x(2x	+1)	+2(2x	+1)
(3x)(2x)	+(3x)(1)	+(2)(2x)	+(2)(1)
= 6x	+3x	+4x	+2
Product of **F**irst terms	Product of **O**uter terms	Product of **I**nner terms	Product of **L**ast terms

Final answer: $6x^2+7x+2$

7:1 Definition of Polynomials

To multiply two polynomials that have more than two terms, multiply each term of one polynomial with each term of the other polynomial. You may multiply vertically or horizontally. See the example below:

7:1 Definition of Polynomials

Example 4: Multiply (3x + 2) with (2x + 1).

Vertical format 3x + 2
2x + 1
3x + 2 1(3x + 2)
$6x^2 + 4x$ 2x(3x + 2)
$6x^2 + 7x + 2$ Product simplified

Horizontal format (3x + 2)(2x + 1)
= 3x(2x + 1) + 2(2x + 1)
= $6x^2 + 4x + 3x + 2$ = $6x^2 + 7x + 2$ Final product

7:1 Definition of Polynomials

Let's review some special products. It may be helpful to just memorize the special products as it will save time since you don't see them frequently

Special Products

Binomial Squares - $(a+b)^2 = a^2 + 2ab + b^2$

$(a-b)^2 = a^2 - 2ab + b^2$

Difference of Squares - $(a+b)(a-b) = a^2 - b^2$

Binomial Cubes - $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

7:3 Graphs of Polynomial Functions

A polynomial function looks like this…

$f(x) = 3x^3 + 2x^2 + x + 1$

The coefficients are not equal to zero.

We have discussed how to graph polynomials of degree 0, 1, and 2.

function	degree	graph
f(x) = 3	0	horizontal line
f(x)=2x+1	1	straight line
$f(x) = 2x^2 + 3x + 1$	2	parabola

7:3 Graphs of Polynomial Functions

Other graphs of degree greater than 2 are more of a challenge to graph, but you can learn to recognize the shapes of these functions. Generally point-plotting is the most accurate way to graph these higher degree functions.

7:3 Graphs of Polynomial Functions

Points to remember about graphs of polynomial functions

- The graph of a polynomial of degree n, may cross the x-axis at most n times.
- The curves of a polynomial function are usually smooth and continuous.
- If the leading coefficient of the function is greater than zero, the graph will rise to the right, otherwise, when the coefficient is less than zero, the graph will fall to the right.