| Name | Period | Date | |
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| | | | |

8:5 *Completing the Square*

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Now that you can graph a quadratic equation, you will learn in this section how to solve the quadratic equation by *completing the square*.

What does it mean "to solve the quadratic equation?" The solution of the quadratic equation is the point (or points) where the parabola crosses the x-axis. (It is possible that the graph will not cross the x-axis). How do you know how many solutions exist? You can determine this by substituting the appropriate values from $y = ax^2 + bx + c$ into the equation $b^2 - 4ac$. If $b^2 - 4ac > 0$, you will have two solutions; if $b^2 - 4ac = 0$, there will be one solution. What happens if $b^2 - 4ac < 0$? There is no real number solution.

To understand what we mean by "completing the square", let's look at how we "expand a binomial" by squaring it.

square of the binomial

perfect-square trinomial

$$(x+2)^2$$
 = x^2+4x+4
 $(x-3)^2$ = x^2-6x+9

If you notice from these examples, the constant term is the square of $\frac{1}{2}$ of the coefficient of the linear term (x-term). In $x^2 + 4x + 4$, 4 is the square of $\frac{4}{2} = 2$. In $x^2 - 6x + 9$, 9 is the square of $\frac{-6}{2} = -3$.

In general terms, we write this as:
$$x^2 + bx + \left(\frac{b}{2}\right)^2$$
, the perfect square, and when we look at the binomial that equals this, it is $\left(x + \frac{b}{2}\right)^2$. So to make an equation $x^2 + bx$ a perfect square, you need to add $\left(\frac{b}{2}\right)^2$ to the expression.

Recall that when you add something to one side of an equation, you must also add the same quantity to the other side. This keeps your equation "equal."

Example 1: Completing the Square.

 $x^2 - 6x - 3 = 0$ by completing the square. Solve:

Solution:

Name

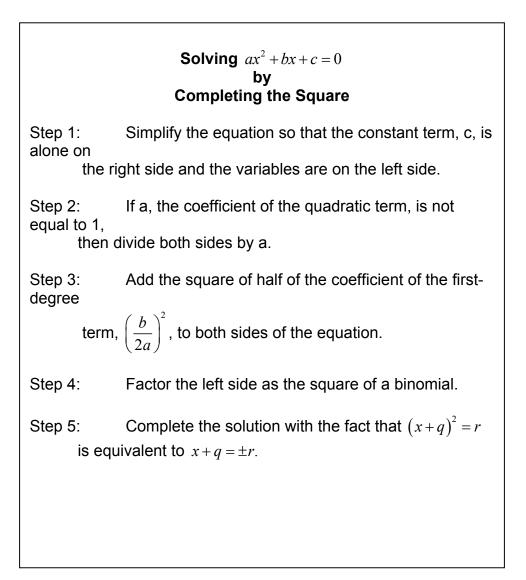
- $x^2 6x 3 = 0$ **Original Equation**
- $x^2 6x = 3$ add -3 to both sides to isolate the variables
- $1\Box x^2 6x = 3$ check the coefficient of x^2 . If it is 1, you are ready to solve. Otherwise, you divide by the coefficient.
- $x^{2} 6x + (-3)^{2} =$ add the square of half of the coefficient of the linear
- $3 + (-3)^2$ term to both sides.
- $x^2 6x + 9 = 12$ simplify
- $(x-3)^2 = 12$ factor the left side as the square of a binomial
- $x 3 = \pm \sqrt{12}$ take the square root of each side.

 $x = 3 \pm 2\sqrt{3}$ simplify

The solutions of the quadratic equation are $3-2\sqrt{3}$ and $3+2\sqrt{3}$. When you approximate the values of the radical, this will give the x-values of the intersection of the parabola with the x-axis of the graph.

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Example 2: Completing the Square

| Solve: | $3x^2 - 6x - 3 = 0$ | |
|-----------|--|---|
| Solution: | $3x^2 - 6x - 3 = 0$ | Original equation |
| | $3x^2 - 6x = 3$ | add 3 to both sides |
| | $x^2 - 2x = 1$ | divide by 3 |
| | $x^{2} - 2x + \left(\frac{-2}{2}\right)^{2} = 1 + \left(\frac{-2}{2}\right)^{2}$ | add the square of half the first- degree term to both sides. |
| | $x^2 - 2x + 1 = 2$ | simplify |
| | $\left(x-1\right)^2=2$ | factor the left side |
| | $x-1 = \pm \sqrt{2}$ | take the square root of each side |
| | $x = 1 \pm \sqrt{2}$ | x = 2.414 and414 |