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8:5 Completing the Square

Now that you can graph a quadratic equation, you will learn in this section how to solve the quadratic equation by *completing the square.*

What does it mean "to solve the quadratic equation?" The solution of the quadratic equation is the point (or points) where the parabola crosses the x-axis. (It is possible that the graph will not cross the x-axis). How do you know how many solutions exist? You can determine this by substituting the appropriate values from $y = ax^2 + bx + c$ into the equation $b^2 - 4ac$. If $b^2 - 4ac > 0$, you will have two solutions; if $b^2 - 4ac = 0$, there will be one solution. What happens if $b^2 - 4ac < 0$? There is no real number solution.

To understand what we mean by "completing the square", let's look at how we "expand a binomial" by squaring it.

square of the binomial		**perfect-square trinomial**
$(x+2)^2$	$=$	$x^2 + 4x + 4$
$(x-3)^2$	$=$	$x^2 - 6x + 9$

If you notice from these examples, the constant term is the square of $\frac{1}{2}$ of the coefficient of the linear term (x-term). In $x^2 + 4x + 4$, 4 is the square of $\frac{4}{2} = 2$. In $x^2 - 6x + 9$, 9 is the square of $\frac{-6}{2}$ = -3.

In general terms, we write this as: $x^2 + bx + \left(\frac{b}{2}\right)^2$, the perfect square, and when we look at the binomial that equals this, it is $\left(x + \frac{b}{2}\right)^2$. So to make an equation $x^2 + bx$ a perfect square, you need to add $\left(\frac{b}{2}\right)^2$ to the expression.

Recall that when you add something to one side of an equation, you must also add the same quantity to the other side. This keeps your equation "equal."

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Example 1: Completing the Square.

Solve: $x^2 - 6x - 3 = 0$ by completing the square.

Solution:	$x^2 - 6x - 3 = 0$	Original Equation
	$x^2 - 6x = 3$	add –3 to both sides to isolate the variables
	$1 \cdot x^2 - 6x = 3$	check the coefficient of x^2. If it is 1, you are ready to solve. Otherwise, you divide by the coefficient.
	$x^2 - 6x + (-3)^2 =$	add the square of half of the coefficient of the linear
	$3 + (-3)^2$	term to both sides.
	$x^2 - 6x + 9 = 12$	simplify
	$(x-3)^2 = 12$	factor the left side as the square of a binomial
	$x - 3 = \pm\sqrt{12}$	take the square root of each side.
	$x = 3 \pm 2\sqrt{3}$	simplify

The solutions of the quadratic equation are $3 - 2\sqrt{3}$ and $3 + 2\sqrt{3}$. When you approximate the values of the radical, this will give the x-values of the intersection of the parabola with the x-axis of the graph.

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Solving $ax^2 + bx + c = 0$
by
Completing the Square

Step 1: Simplify the equation so that the constant term, c, is alone on
the right side and the variables are on the left side.

Step 2: If a, the coefficient of the quadratic term, is not equal to 1,
then divide both sides by a.

Step 3: Add the square of half of the coefficient of the first-degree
term, $\left(\frac{b}{2a}\right)^2$, to both sides of the equation.

Step 4: Factor the left side as the square of a binomial.

Step 5: Complete the solution with the fact that $(x+q)^2 = r$
is equivalent to $x + q = \pm r$.

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Example 2: Completing the Square

Solve: $3x^2 - 6x - 3 = 0$

Solution: $3x^2 - 6x - 3 = 0$ Original equation

$3x^2 - 6x = 3$ add 3 to both sides

$x^2 - 2x = 1$ divide by 3

$x^2 - 2x + \left(\frac{-2}{2}\right)^2 = 1 + \left(\frac{-2}{2}\right)^2$ add the square of half the first-degree term to both sides.

$x^2 - 2x + 1 = 2$ simplify

$(x-1)^2 = 2$ factor the left side

$x - 1 = \pm\sqrt{2}$ take the square root of each side

$x = 1 \pm \sqrt{2}$ x = 2.414 and -.414