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## *8:1 Solving Quadratic Equations by Finding Square Roots*

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# QUADRATIC EQUATIONS AND PARABOLAS

In this chapter, you will study:

- Solving Quadratic Equations by Finding Square Roots
- Graphs of Quadratic Equations: Parabolas
- Solving Quadratic Equations by Completing the Square
- The Quadratic Formula
- Imaginary and Complex Numbers
- Solving any Quadratic Equation

A quadratic equation is one that can be written as  $ax^2 + bx + c = 0$ , which is known as the standard form of the equation where  $a$ ,  $b$ , and  $c$  are real numbers and  $a \neq 0$ . ( $a$  is the *leading coefficient* – the number associated with the quadratic term).

From Algebra 1, you should remember that positive real numbers have two square roots,  $\sqrt{b}$  and  $-\sqrt{b}$ , the positive and the negative, which are sometimes designated by the sign  $\pm\sqrt{b}$ . Recall that square roots are indicated by the radical symbol,  $\sqrt{\quad}$ , and the number under the radical sign is called the radicand.

## 8:1 Solving Quadratic Equations by Finding Square Roots

**Finding the  $n$ th root of  $b$** 

1. An  $n$ th root of  $b$  is a solution of the equation  $x^n = b$ .
2.
  - a. If  $n$  is even and  $b > 0$ , there are two roots of  $b$ .  
The principal (or positive)  $n$ th root of  $b$  is denoted  $\sqrt[n]{b}$ .  
The other root of  $b$  is denoted  $-\sqrt[n]{b}$ .
  - b. If  $n$  is even and  $b = 0$ , there is one  $n$ th root:  $\sqrt[n]{0} = 0$ .
  - c. If  $n$  is even and  $b < 0$ , there is no real  $n$ th root of  $b$ .
3. If  $n$  is odd, there is exactly one real  $n$ th root of  $b$ , whether  $b$  is positive, negative, or zero.

Example 1: Solving a Quadratic Equation

Solve:

a.  $x^2 = 9$

b.  $5x^2 = 15$

Solution:

a.  $x^2 = 9$

$x = \pm\sqrt{9} = \pm 3$

The roots are 3 and  $-3$ 

b.  $5x^2 = 15$

$x^2 = 3$

$x = \pm\sqrt{3}$

The roots are  $\sqrt{3}$  and  $-\sqrt{3}$

**8:1 Solving Quadratic Equations by Finding Square Roots**

A cube root of a number is a solution of the equation  $x^3 = b$ . Every number  $b$ , whether positive, negative, or zero has exactly one real cube root, denoted  $\sqrt[3]{b}$ . The cube root of a positive number is positive and the cube root of a negative number is negative.

Example 2. Simplify.

a.  $\sqrt[3]{8}$                       b.  $\sqrt[3]{-27}$                       c.  $\sqrt[3]{10^6}$

Solution:

- a.  $\sqrt[3]{8} = 2$  because  $2^3 = 8$   
b.  $\sqrt[3]{-27} = -3$  because  $(-3)^3 = -27$   
c.  $\sqrt[3]{10^6} = 10^2 = 100$  because  $(10^2)^3 = 10^6$

Example 3. Simplify.

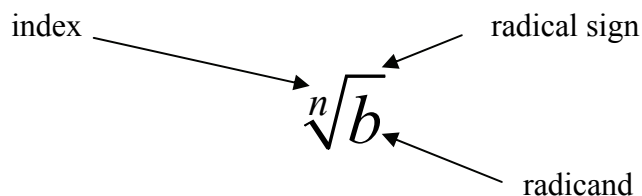
a.  $\sqrt{81}$                       b.  $\sqrt[3]{27}$                       c.  $\sqrt[5]{-32}$                       d.  $\sqrt[4]{-1}$

Solution:

- a.  $\sqrt{81} = \pm 9$  since  $9^2 = 81$  and  $(-9)^2 = 81$   
b.  $\sqrt[3]{27} = 3$  since  $3^3 = 27$   
c.  $\sqrt[5]{-32} = -2$  since  $(-2)^5 = -32$   
d.  $\sqrt[4]{-1} = \text{no solution}$ . There is no real number raised to the 4<sup>th</sup> power equal to  $-1$ .

### 8:1 Solving Quadratic Equations by Finding Square Roots

The symbol  $\sqrt[n]{b}$  is called a radical. Each part of the radical has a name. See below:



When there is no number for an index, it is understood that it is a square root. It is customary not to write the 2 in the radical sign.

Properties of Radicals	Examples
1. $(\sqrt[n]{b})^n = b$ because $\sqrt[n]{b}$ satisfies $x^n = b$	$(\sqrt{5})^2 = 5$  $(\sqrt[3]{-5})^3 = -5$
2. $\sqrt[n]{b^n} = b$ if n is odd	$\sqrt[3]{6^3} = 6$  $\sqrt[5]{x^5} = x$
3. $\sqrt[n]{b^n} =  b $ if n is even, because the principal <i>n</i> th root is always non-negative for even values of <i>n</i> .	$\sqrt{(-5)^2} =  -5  = 5$  $\sqrt{(x-2)^2} =$ $ x-2  =$ $x-2, x > 2$ $2-x, x < 2$